Logic 2 Homework 1

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Feel free to cite results from your lecture notes, the official lecture notes, or textbooks referenced on the course website. You are also allowed and encouraged to collaborate with your classmates, but you must write your own solutions. Please do not use any other sources without first discussing with the instructor.

Fraïssé Theory

Problem 1.1. Construct a graph on vertex set $V = \{v_i : i \in \mathbb{N}\}$ so that if $i < j$, vertices v_i and v_j are adjacent if and only if the bit representing 2^i in the binary representation of j is 1.
Show that this graph is a Fraïssé limit of the class of all finite graphs.

Problem 1.2. Show that the class of all finite *triangle-free* graphs is a Fraïssé class.

Problem 1.3. Show that the class of finite equivalence relations (in the language $\{E\}$ of one binary relation symbol for equivalence) is Fraïssé, and its Fraïssé limit is the equivalence relation with $\aleph_0$ many $\aleph_0$-sized classes.

Problem 1.4. If K is a finite field, show that the class of finite-dimensional K-vector spaces is Fraïssé, the $\aleph_0$-dimensional K-vector space is its Fraïssé limit.
Give (or at least describe) a simple axiomatization of the complete theory of this Fraïssé limit, and show that it is $\aleph_0$-categorical.

Problem 1.5. Show that the class of finite boolean algebras (in the language $\{\top, \bot, \wedge, \vee, \cdot^c\}$) is Fraïssé, and it has a Fraïssé limit, the unique countable atomless boolean algebra.
Give (or at least describe) a simple axiomatization of the complete theory of this Fraïssé limit, and show that it is $\aleph_0$-categorical.

Compactness Review

Problem 1.6. Prove the Compactness theorem by following the proof in the following exercises of Marker, Chapter 2:

- 2.5.18
- 2.5.19
- 2.5.20

See the course website for information on obtaining Marker's Model Theory textbook.