Logic 2 Homework 3

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Feel free to cite results from your lecture notes, the official lecture notes, or textbooks referenced on the course website. You are also allowed and encouraged to collaborate with your classmates, but you must write your own solutions. Please do not use any other sources without first discussing with the instructor.

Problem 3.1. Given $D \subseteq M^{m+n}$ definable in an ordered structure $\mathcal{M}$, show that

- $\{\bar{a} \in M^m : D_{\bar{a}} \text{ is open}\}$
- $\{(\bar{a}, \bar{b}) \in M^{m+n} : \bar{b} \in \mathsf{int}(D_{\bar{a}})\}$

are both definable.

Problem 3.2. Assume $\mathcal{M}$ is o-minimal.

- Show that if $D \subseteq M$ is definable and infinite, it contains an open interval.
- Show that if $D \subseteq M^{m+1}$, then $\{\bar{a} \in M^m : D_{\bar{a}} \text{ is finite}\}$ is definable.

Problem 3.3.

- The image of a definably connected $D \subseteq M^m$ under a continuous function $f : D \to M^n$ whose graph is definable is definably connected.
- If $X \subseteq M^m$ is definably connected and $X \subseteq Y \subseteq \mathsf{cl}(X)$ with Y definable, then Y is definably connected.
- If $X, Y \subseteq M^m$ are definably connected and $\mathsf{cl}(X) \cap Y \neq \emptyset$, then $X \cup Y$ is definably connected.
- If $\mathcal{M}$ is definably complete (for instance, if it is o-minimal), then the definably connected sets $D \subseteq M$ are precisely intervals (with any endpoint behavior, including $\pm\infty$).

Exercises from Marker:

Problem 3.4. Give a model-theoretic proof of the full version of Chevalley's Theorem: For any multivariable polynomial $p : K^m \to K^n$, if $D \subseteq K^m$ is constructible, then so is its image $p(D)$.

Exercises from van den Dries:

Problem 3.5. Let $\mathcal{M} \models \mathsf{ODAG}$. Show that $+ : M^2 \to M$ and $- : M \to M$ are continuous with respect to the order topology.

Problem 3.6. Let $\mathcal{M}$ be an ordered division ring. Then $\mathcal{M} \models \mathsf{DLO}$ (prove this or assume this). Show that $\times : M^2 \to M$ and $\cdot^{-1} : M \setminus \{0\} \to M$ are continuous with respect to the order topology.