Logic 2 Homework 5

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Feel free to cite results from your lecture notes, the official lecture notes, or textbooks referenced on the course website. You are also allowed and encouraged to collaborate with your classmates, but you must write your own solutions. Please do not use any other sources without first discussing with the instructor.

Problem 5.1 ((Exercise 5.1.8)). If C is a cell, and we let $\pi : M^{m+n} \to M^m$ be the projection, then both the projection $\pi(C)$ is a cell and every fiber $C_{\bar{a}} \subseteq M^n$ is a cell of the same dimension. Furthermore, for any $\bar{a} \in \pi(C)$, $\dim(C) = \dim(\pi(C)) + \dim(C_{\bar{a}})$.

Problem 5.2 ((Exercise 5.1.11)). If C is a cell, construct an explicit cell decomposition for $\mathrm{cl}(C)$ and use it to show that $\dim(\partial C) < \dim(C)$.

Problem 5.3 ((Exercise 5.3.2)). Suppose $\mathcal{M} \vDash \mathsf{VS}_K$ for some field K. Then for $A \subseteq M$,

$$\mathrm{dcl}(A) = \mathrm{acl}(A) = \mathrm{Span}_K(A).$$

Problem 5.4 ((Exercise 5.3.3)). Suppose $\mathcal{M} \vDash \mathsf{ACF}$. Then for $A \subseteq M$, $\mathrm{acl}(A)$ is the algebraic closure of the subfield generated by A.

Problem 5.5 ((Exercise 5.3.7)). Prove that $\forall A \subseteq X, \mathrm{dcl}(\mathrm{dcl}(A)) = \mathrm{dcl}(A)$, and that both $\mathrm{dcl}, \mathrm{acl}$ are finitary.